Universality in Random Systems: the case of the 3-d Random Field Ising model

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ABSTRACT

We study numerically the zero temperature Random Field Ising Model on cubic lattices of various linear sizes $6 \leq L \leq 90$ in three dimensions with the purpose of verifying the validity of universality for disordered systems. For each random field configuration we vary the ferromagnetic coupling strength J and compute the ground state *exactly*. We examine the case of different random field probability distributions: gaussian distribution, zero width bimodal distribution $h_i = \pm 1$, wide bimodal distribution $h_i = \pm 1 + \delta h$ (with a gaussian δh). We also study the case of the randomly-diluted antiferromagnet in a field, which is thought to be in the same universality class. We find that in the infinite volume limit the magnetization is discontinuous in J and we compute the relevant exponent, which, according to finite size scaling, equals $1/\nu$. We find different values of ν for the different random field distributions, in disagreement with universality.

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It is well known in the theory of phase transitions that critical exponents of very different physical systems can be identical. This property is called universality. Universality is one of the most important concepts in modern physics and can be explained in the framework of the perturbative renormalization group, where it is shown that critical exponents depend only on very few “relevant” parameters of the physical system (dimentionality of space, dimentionality of the order parameter, symmetry properties, ...) and do not depend on the details of the interactions.

It is generally assumed that universality is also valid in the presence of quenched disorder. The theoretical justification requires, in this case, the use of the replica trick, in addition to the perturbative renormalization group(PRG), and is therefore weaker in the case of disordered systems. Furthermore it is known that PRG fails in the case of the random field Ising model (RFIM)[1]. The RFIM is (together with branched polymers[2]) one of the very few cases where PRG can be analyzed to all orders of pertubation theory[3, 4] and this analysis leeds to false conclusions (while for the branched polymers the same analysis is correct!).

Universality is difficult to check because the difference of the critical exponents in different systems is in general small. Different attempts to check universality in disordered Potts models or diluted ferromagnets in two dimensions gave conflicting results. The purpose of this paper is to check universality in the case of the RFIM in three dimensions using numerical simulations. In a previous paper[5] to be referred in the following as **I** , we studied the cases of the gaussian and bimodal distribution of random fields and we found evidence that universality is violated in the RFIM. In this paper we present also the cases of the diluted random antiferromagnet in a field (DRAF) which is believed to be in the same universality class and of the wide bimodal distribution (see below). The evidence for violation of universality becomes much stronger.

We would like to mention the very recent preprint by Ballesteros et al.[6] on three dimentional diluted Ising ferromagnets. Theese autors measure the critical exponents and universal ratios for different dilutions. Their results are dilution independent, in agreement with universality, provided that they take into account the first nonleading correction to scaling.

The Hamiltonian of the RFIM is given by

$$H \; = \; -J \sum_{<i,j>} \sigma_i \sigma_j \; - \; \sum_i \; h_i \sigma_i \tag{1}$$

where $\sum_{<i,j>}$ runs over neighbouring sites of the lattice (we have only considered three dimensional cubic lattices with periodic boundary conditions) and h_i are independent random variables identically distributed with probability distribution $P(h)$. In the following we will consider the case of zero mean gaussian distribution $P(h) = \frac{1}{\sqrt{2\pi}} \exp -h^2/2$, of the bimodal distribution $P(h) = \frac{1}{2}(\delta(h-1) + \delta(h+1))$, and of the wide bimodal distribution $P(h) = \frac{1}{2w\sqrt{2\pi}}(\exp -(h-1)^2/2w^2 + \exp -(h+1)^2/2w^2)$. We also considered the case of the deleted random antiferromagnet in a field H which is believed to be in the same universality class. When the ratio H/J is rational, additional singularities appear in the distribution of the ground states. For this reason we considered a random field in this case also, with probability distribution $P(h) = \frac{1}{w\sqrt{2\pi}} \exp -(h-1)^2/2w^2$. It is straitforward to show in the framework of PRG

that the addition of a random component does not change the universality class. We considered the case of site dilution with probability $p = .40$ and w=.1

For a given random field sample, one can vary both the ferromagnetic coupling J and the temperature T, i.e. the phase boundary is a line in the J, T plane. It is thought, in accordance with PRG, that the nature of the transition and the value of the exponents do not depend on the position on the transition line, nor on the direction one crosses it, and that this is true down to zero temperature. Furthermore it has been argued that the renormalization group flow drives the system to $T = 0$, i.e that $T = 0$ is an attractive fixed point of the renormalization group. So it is advantageous to work at $T = 0$ where it has been shown that the RFIM is equivalent to the problem of maximum flow in a graph[7], for which very fast (polynomial) algorithms are known. Another advantage of this algorithm is that it provides the exact groud state and therefore there is no thermalization problem. Simulations using such algorithms have already been performed in the past[8, 9]. In the present paper we use the latest version of the algorithm developped by Goldberg and Tarjan[10], which we optimized for the case of the cubic lattice. It has been shown that this algorithm converges to the ground state in a time $t < L^6 \ln L$ where L is the linear size of the cubic lattice. We found experimentally that $t \sim L^4$[11].

We proceeded as follows: We first produced a configuration of the external fields. We then chose k values of the ferromagnetic coupling $J_1, J_2, ..., J_k$ and we computed the corresponding ground states. We considered lattices of different linear sizes L, from $L = 6$ to $L = 90$. We studied between 1000 samples for $L = 90$ and 40000 samples for $L = 6$. We studied the variation of the absolute value of the magnetization $m(J)$ and of the energy as a function of J. We found that there is a region in J where there are large discontinuities of $|m|$ and that outside that region $m(J)$ is a smooth function of J. The amplitude of the discontinuities is volume independent, while the width of the region in J where they appear, shrinks as the volume increases. For every $\{h_i\}$ sample, we chose the n_d largest variations of the magnetization between two succesive values of J. Let's call $j_1 < j_2 < \cdots < j_{n_d}$ the values of J at which they occur. The choice of n_d is somehow arbitrary. We took $2 \leq n_d \leq 6$. Our results are compatible with the hypothesis

$$j_i \;=\; j_\infty \;+\; \frac{c_i}{L^p}\;(1 \;+\; \frac{f_i}{L^q}\;) + \delta j_i \tag{2}$$

δj_i are zero mean gaussian random variables with a width σ decreasing with the volume

$$\sigma(L) \sim \sigma_0 L^{-\delta}(1 + \sigma_1 L^{-\rho}) \tag{3}$$

As the volume increases the mean is shifted towards the infinite volume value j_∞, which is the same for all the j_i's and at the same time the variance of the distributions shrinks to zero. This shift of the "critical coupling" is analogous to the shift of the effective critical temperature due to finite volume corrections, well known to occur in ordinary second order phase transitions. We conclude that the appearance of several discontinuities in the magnetization is a finite volume artifact and that our results are fully compatible with the hypothesis of a single discontinuity in the thermodynamic limit. This is true for all distributions of the random field we studied. Figure 1 shows

the average absolute value of the magnetization difference Dm_2 as a function of $1/L$ for the case of the DRAF and for the wide bimodal distribution and Dm_4 for the gaussian case, where $Dm_k = \overline{|m(j_k)^+ - m(j_1)^-|}$, $m(j_k)^+ = \lim_{j\to j_k,\ j>j_k} m(j)$ and $m(j_1)^- = \lim_{j\to j_1,\ j<j_1} m(j)$, i.e. Dm_k is the average magnetization difference computed between the k largest discontinuities.

These discontinuities have not been seen in the previous simulations for the following reason. Only the average magnetization has been measured and for finite volumes, the position of the dicontinuities fluctuates from sample to sample so that the average magnetization is continuous. This raises the question of how to average over the disorder. The only method available for analytic calculations is the replica method. With this method one can compute the averages of observables for fixed values of the couplings. This in turn has inspired the numerical work. But there is more freedom with numerical simulations, as we have illustrated above. We see in the present case that averaging the magnetization for fixed values of the ferromagnetic coupling, hides the true nature of the transition, i.e. the discontinuity of the magnetization.

In figure 2 we plot $Dj = \overline{j_2 - j_1}$, i.e. the average difference of the values of the couplings at which the two largest discontinuities of the magnetization occur, versus $1/L$. The continuous lines are the best fit to the data. The upper line is for the DRAF and the lower line for the wide bimodal distribution. The interested reader can find the data for the gaussian distribution in **I** . We conclude that the ansatz of equation (4) describes well the data down to a value of L as small as $L = 6$. To be more quantitatif, the χ^2 per degree of freedom is $\chi^2 = .9$ for the gaussian case, $\chi^2 = 1.1$ for the DRAF and $\chi^2 = 2.75$ for the wide bimodal distribution.

Let's now discuss some possible origins of systematic errors. An obvious one is the choice of n_d, the number of discontinuities we choose to analyse. The number of large discontinuities fluctuates from sample to sample and the choice of n_d is not obvious. We have choosen the largest value of n_d which is compatible with the data, i.e. provides a χ^2 per degree of freedom of the order of one. This is $n_d = 5$ for the gaussian distribution and $n_d = 2$ for the other distributions. We studied the dependence of the exponents on n_d and we found it to be small. A detailed discussion for the gaussian distribution is found in **I** . Another important point is the choice of the values of the ferromagnetic couplings $J_1 < J_2, \cdots, < J_m$ at which we compute the ground states. The values j_i at which the strong discontinuities of the magnetization occur, strongly fluctuate from sample to sample. So if the total range of J_1 to J_m is not large enough, we may miss some of the discontinuities. On the other hand the J_k's must be dense enough in order to be able to observe a discontinuity. The obvious way to satisfy both requirements is to compute a very large number of ground states per sample but this is impossible because of computer time limitations. For the gaussian distribution (see **I**) we chose equally spaced $x = 1/J$'s with $\delta x = .0125$ and required that for every size L, $J_1 < \overline{j}_1 - 4\sigma_1$ where $\overline{j}_1$ is the average location of the first sigularity for that size and σ_1 its variance. We required similarly that $J_m > \overline{j}_n + 4\sigma_n$ where j_n is the last singularity. For the wide bimodal distribution and the DRAF we proceeded differently. Let the ground state energy for the coupling J be $-E(J)$ and the corresponding value of the "exchange energy" be $E_e(J) = \sum_{<i,j>} \sigma_i\sigma_j$. It can be shown that $E(J)$ is a convex function of J. If we know $E(J)$ and its derivative $E_e(J)$ for $J_1 < J_2 \cdots < J_k$ then we have, because of the convexity, both an upper and a lower bound for $E(J)$ for $J_1 < J < J_k$. We start by

computing a few ground states (small k) sparsely sampling a very large J region. Then we compute the values of J at which the difference of the upper and the lower bound is maximum and we compute the ground states for those values of the J. We iterate the procedure until the difference between the upper and the lower bound is everywhere smaller than $5 \times 10^{-6} \times E(J)$. We further require that the resolution in J around the stronger discontinuities be better than 10^{-4}. This is an efficient way to cover a large domain in J, while concentrating computational effort where $E(J)$ and $m(J)$ vary the most.

Despite the magnetization discontinuity, we do not think that this is a first order phase transition for two reasons: the non-classical values of the exponents and the continuity of the energy derivative. It was shown in **I** that the discontinuities of the energy derivative, which appear at the same value of the coupling as the magnetization discontinuities, vanish in the infinite volume limit. In the case of the bimodal distribution, additional energy discontinuities appear at rational values of the couplings. These discontinuities do not disappear in the infinite volume limit. We found that they disappear if we add a small width to the field distribution. A similar phenomenon is observed for the DRAF. This is the reason we introduced the wide bimodal distribution and we added a gaussian width to the field in the diluted antiferromagnet case.

Figure 3 shows the values of the exponents p and q that are compatible with our data (within 90% confidence) for the different random field distributions. We see that the values of p and q are correlated and the uncertainties on p and q are quite large. Previous simulations were unable to determine the first correction to scaling, because of the limited number of sizes and of statistical errors, so the data were fitted to a single power law. We found that the two different fitting assumptions, a single power law behaviour or the inclusion of a subleading correction, lead to quite different values of the exponent $1/\nu$ and drive it considerably out of the statistical error bars. For a detailed discussion and comparison with previous work see **I**.

We also determined the exponents δ and ρ which measure sample to sample fluctuations (see equation (3)). We find that these exponents are identical to p and q within our error bars. A hand waiving argument to explain this is to say that the appearence of several singularities, the shift of the critical value of the coupling and the sample to sample fluctuations are finite size effects which are controlled by the "dimensions of the same operators". We feel that this argument has to be substanciated. We find this result remarkable and consider it as a confirmation of our analysis.

According to finite size scaling, $p = 1/\nu$ and q should be universal, i.e. the same for all probability distributions of the random field. We see that our data are compatible with the hypothesis that the random diluted antiferromagnet in a field and the gaussian random field are in the same universality class. This is a highly nontrivial result of perturbative renormalization group, because the two systems are completely different. The bimodal and wide bimodal distributions are also mutually compatible for $q >$ 3. This is not visible in figure 3. But if one takes into account all the probability distributions of the random fields our data are clearly not compatible with universality, despite the large uncertainties on p and q. This is also confirmed by the following analysis. We can fit the data for the three random field distributions, i.e. the gaussian, the wide bimodal and DRAF, with the assumption of universality, i.e. the assumption that they share the same values of the exponents. We then find $p = .55$, $q = 1.25$

with a χ^2 per degree of freedom $\chi^2 = 11.2$ This is to be compared with $\chi^2 = 1.6$ per degree of freedom for the hypothesis of different exponents. We conclude that our data present strong evidence for the violation of universality in the RFIM at zero temperature, contrary to what is commonly believed.

One may argue that a second nonleading correction to finite site scaling may restore universality. We do not know of any theoretical argument excluding such corrections. But if this additional term is necessary for the determination of critical exponents in disordered systems, then it would be impossible to measure them numerically (and probably also experimentally) in the near future.

After completion of this work we became aware of a preprint by Hartmann and Nowak[12] where they study numerically the ground state properties of random field Ising models in three dimensions.

I would like to thank J.-C. Anglès d'Auriac for his pleasant and fruitfull collaboration in the first part of this work.

This paper is dedicated to Heinz Horner on the occasion of his 60th birthday.

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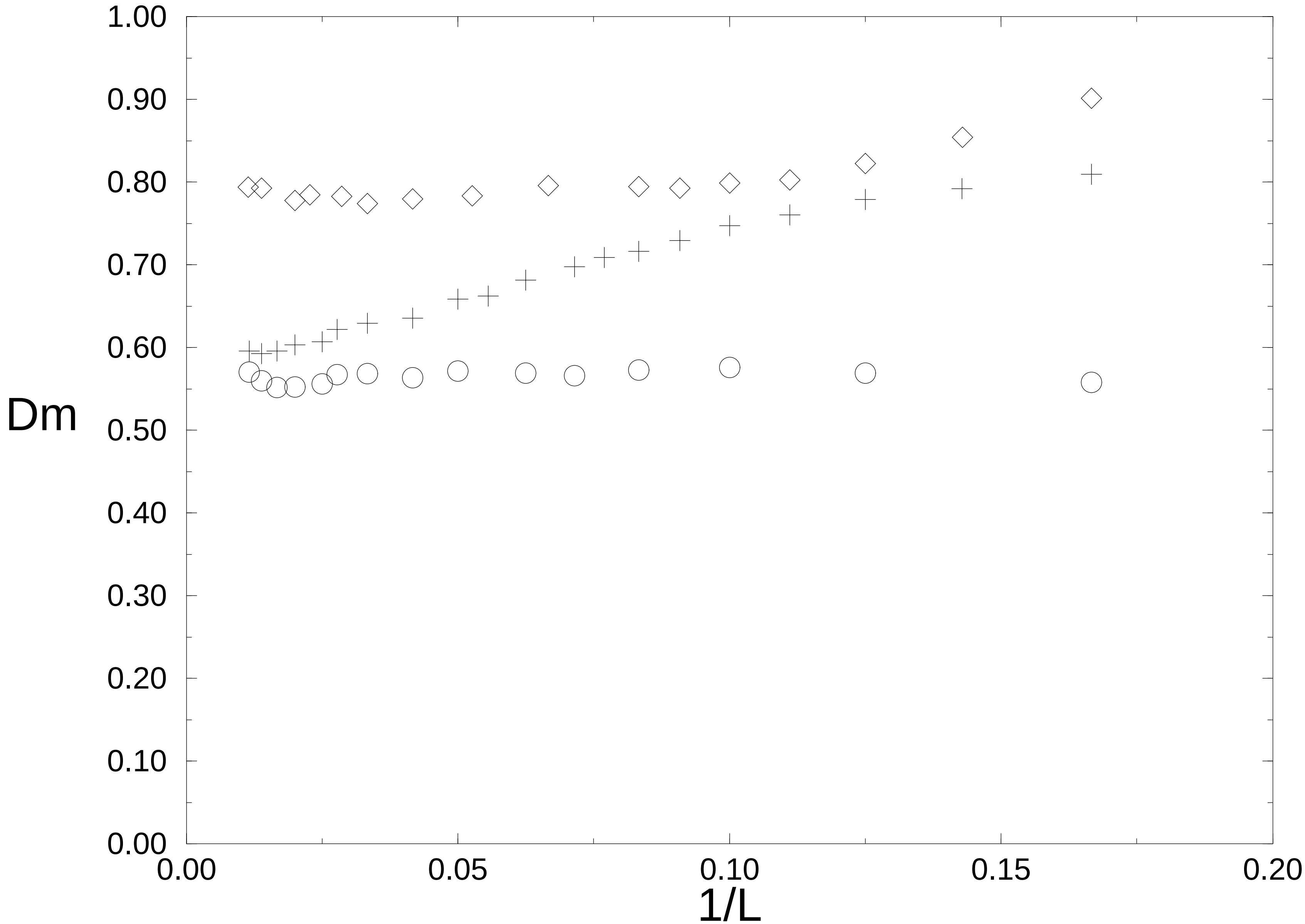
Figure captions

Figure 1. Average magnetization discontinuities as a function of inverse lattice size $1/L$. Diamonds show the data for the gaussian distribution, crosses for the wide bimodal distribution and circles for the randomly diluted antiferromagnet in a field.

Figure 2. Average difference of the values of the couplings for the two largest discontinuities of the magnetization, as a function of inverse lattice size $1/L$. Diamonds show the data for the wide bimodal distribution and circles for the randomly diluted antiferromagnet in a field.

Figure 3. 90% confidence levels for the exponents p and q for the different random field distributions. G denotes the gaussian distribution, AF the diluted antiferromagnet in a field, BM the mimodal distribution and WBM the wide bimodal distribution.



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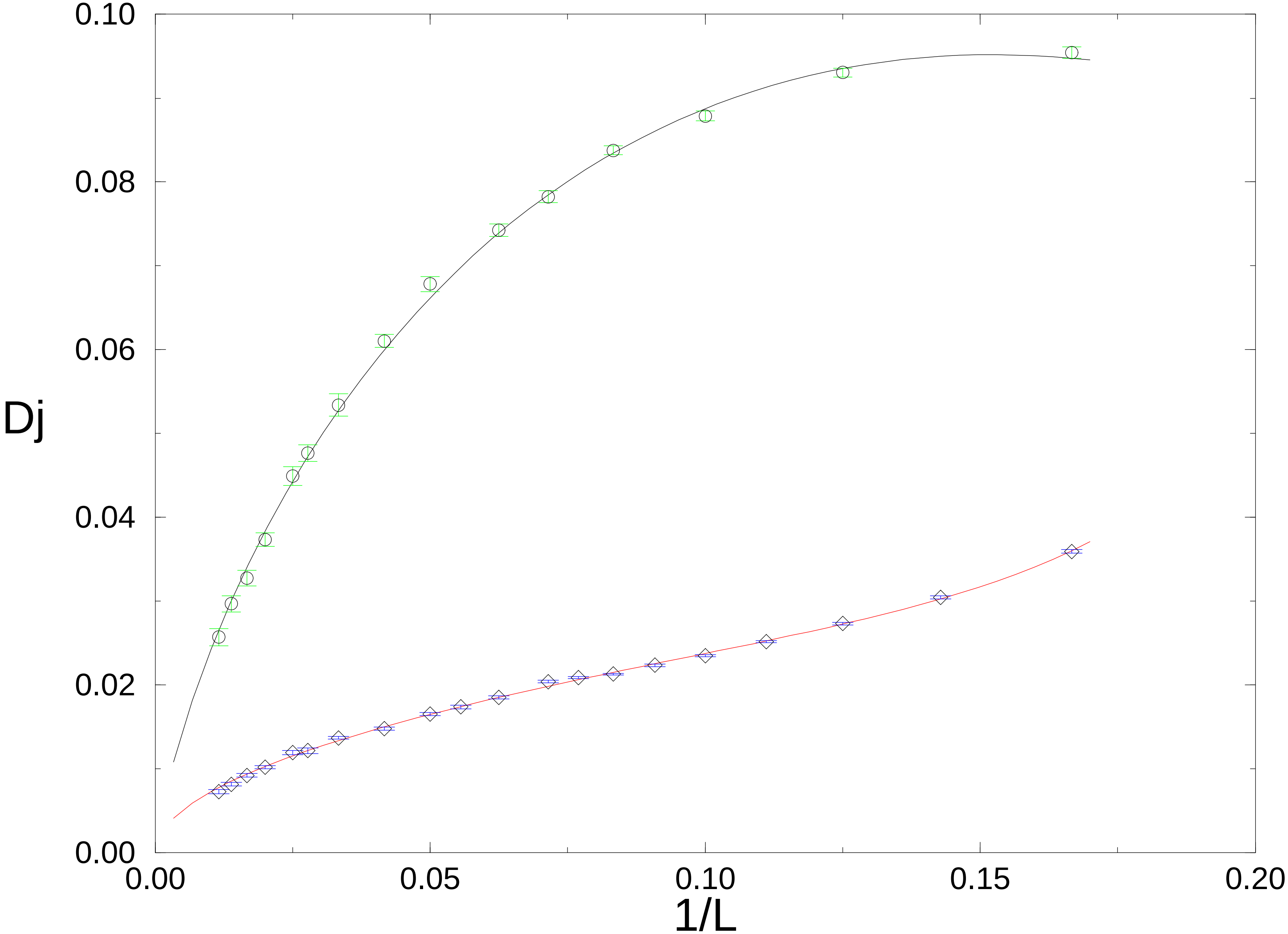
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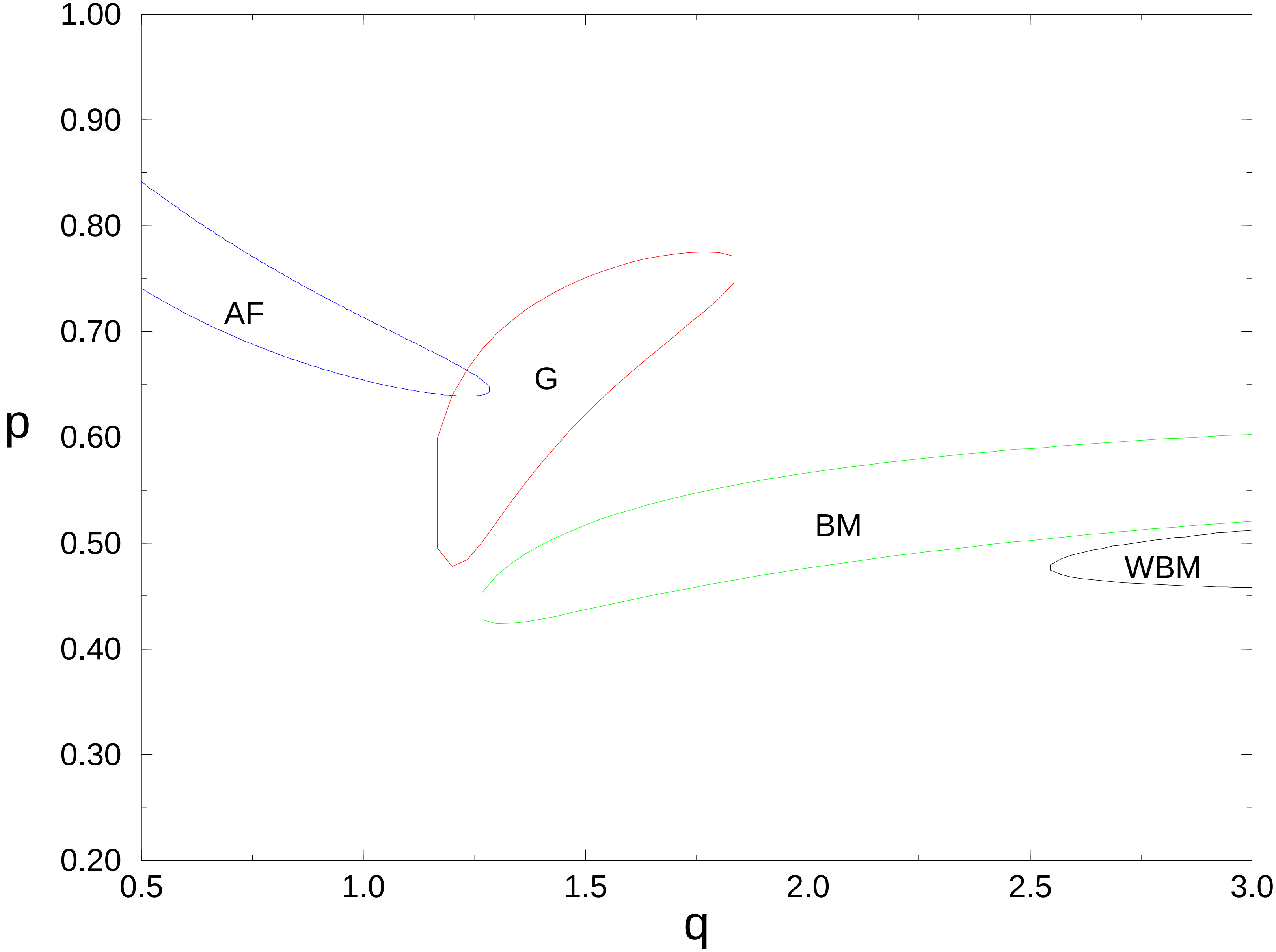

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